Course introduction:

Quantum Field Theory is an indispensable theoretical tool in modern condensed matter physics. The primary goal of the course is intended to be two-folded; one is to get used to the field-theoretical method of quantum many-particle systems and the other is to study macroscopic quantum phenomena such as superconductivity and superfluidity using the field theory. The course assumes that students are well acquainted to quantum mechanics, statistical mechanics, complex analysis and electromagnetism at the undergraduate level. To make it self-contained, a review on the second quantization is given at the first one or two weeks. In the former part of the course, Green's function approaches to quantum many-particle systems are introduced, where standard Feynman-Dyson perturbation theory in terms of diagrams is developed. In the latter part, the method is applied to several condensed matter systems, such as electron-phonon systems, superconductors and superfluid Helium. One of the most relevant and successful applications of the field-theoretic method in condensed matter physics is the Bardeen-Cooper-Schrieffer (BCS) theory, through which we will study electromagnetic and thermodynamic properties of conventional superconductors. If time allows, we might also have a couple of classes about van der Waals force, collective modes in magnets, and electric resistivity in metals.

Requirements:

The course assessment will be comprised of two parts; (1) homework assignment [65%] and (2) final exam [35%].

Syllabus:

1. a review of second quantization

1.1 general introduction

1.2 linear harmonic oscillator

1.3 many harmonic oscillators

1.4 field quantization,

1.5 systems of indistinguishable particles; boson and fermion,

1.6 creation operator and annihilation (destruction) operators

1.7 Hamiltonian and other operators in terms of creation and destruction operators

1.8 degenerate electron gas

2. Field Theory at the zero-temperature

2.1 Schrodinger, Heisenberg, and interaction pictures

2.2 adiabatic switching and Gell-Mann and Low theorem,

2.3 Green's function

2.4 Wick theorem

2.5 diagramatic analysis of perturbation theory (fermion case)

2.6 (self-consistent) Hartree-Fock approximation

2.7 imperfect Fermi gas; dilute gas with short-range interaction, ladder approximation

2.8 degenerate electron gas; high-density gas with long-range Coulomb interaction

3. application to electron gas at the zero temperature

3.1 screening effect in degenerate electron gas,

3.2 collective modes; plasma oscillation and zero sound mode

4. Field Theory at finite temperature

4.1 temperature (Matsubara) Green's function

4.2 temperature Green’s function in the interaction picture

4.3 Wick’s theorem for temperature Green’s function

4.4 Feynman Rule for temperature Green’s function

4.5 Dyson equations and Hartree-Fock approximation

4.6 Specific heat of an imperfect Fermi gas at low-temperature,

4.7 real-time Green's functions and generalized Lehmann representation

4.8 linear response at finite temperature

4.8.1 linear response theory

4.8.2 electric conductivities

4.9 collective modes at high temperature

5. application to electron-phonon systems

5.1 non-interacting phonon system and Debye theory of specific heat

5.2 electron-phonon interaction

5.3 Feynman rule for electron-phonon systems:

equivalent electron-electron interaction and BCS Hamiltonian

6. application to superconductivity

6.1 superconducting properties and thermodynamic relations

6.2 BCS theory; Cooper pair, canonical transformation and gap equation

6.3 electromagnetic properties of superconductors;

6.3.1 paramagnetic and diamagnetic currents

6.3.2 electromagnetic response kernel

6.3.3 Meissner effect; magnetic penetration depth

6.3.4 derivation of Pippard equation

6.3.5 derivation of GL equation

Reference Textbook:

``Statistical Mechanics’’, R.P. Feynman, isbn 0-201-36076-4

``Quantum Theory of Many-Particle Systems’’ Fetter and Walecka, isbn 0-486-42827-3